



## PERFORMANCE COMPARISON OF RAYLEIGH FADING CHANNELS IN MIMO SYSTEMS WITH DIFFERENT ZERO FORCING EQUALIZATION TECHNIQUES

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**Abstract:** The use of multiple antennas for transmission and reception is a major technological advance that is expected to make it possible to increase the data rate in wireless networks by orders of magnitude. While the use of more than one antenna at the receiver side is quite common, only recently was a communication theory of multiple transmit and receive antenna developed. Equalizer design must typically balance Inter Symbol Interference mitigation with noise enhancement, since both the signal and the noise pass through the equalizer, which can increase the noise power. Moreover, equalizers require an estimate of the channel impulse or frequency response to mitigate the resulting Interference Symbol Interference. In this paper the performance comparison of Multiple Input Multiple Output Systems with different Zero Forcing Equalization Techniques using Binary Phase Shift Keying Modulation in Rayleigh Channel is provided.

**Keywords** - Rayleigh, Interference, Multiple antennas, Equalization

### Introduction to Multiple Antennas

Wireless channels suffer from time-varying impairments such as multipath fading, interference, and noise. Diversity, such as time, frequency, space, polarization, or angle diversity, is typically used to mitigate these impairments. Diversity gain is achieved by receiving independent-fading replicas of the signal. The multiple antenna system employs

multiple antennas at either the transmitter or the receiver, and it can be either multiple-input single-output (MISO) for beam forming or transmit diversity at the transmitter, single-input multiple-output (SIMO) for diversity combining at the receiver, or MIMO, depending on the number of transmit and receive antennas. The MISO, SIMO and MIMO channel models can be generated by using the angle-delay scattering function.

Multiple antenna systems are generally grouped as smart antenna systems and MIMO systems. A smart antenna system is a subsystem that contains multiple antennas, based on the spatial diversity and signal processing; it significantly increases the performance of wireless communication

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systems. Direction-finding and beam forming are the two most fundamental topics of smart antennas. Direction-finding is used to estimate the number of emitting source and their DoAs, while beam forming is used to estimate the signal-of-interest (SOI) in the presence of interference.

A MIMO system consists of multiple antennas at both the transmitter and receiver. They are typically used for transmit diversity and spatial multiplexing. Spatial multiplexing can maximize the system capacity by transmitting at each transmit antenna a different bit stream. MISO, SIMO and MIMO can be collectively treated as MIMO, and thus the smart antenna system can be regarded as a special case of the MIMO system [1]. For implementation of multiple antenna systems, a major problem is the knowledge of the channel at the transmitter as well as the receiver. The situation may be such that Channel State Information (CSI) is available at the transmitter and the receiver. The situation may be such that CSI is available at both the receiver and the transmitter, CSI is available at the receiver alone, or no CSI is available [3]. The performance can be improved if more CSI is available. Multiple antennas are difficult to implement on the Mobile Stations due to the size and power restrictions.

### I. Zero Forcing Equalizer

If the channel is ISI distorted by  $H_c(f)$ , the linear equalizer removes the ISI distortion by applying channel inversion,  $H_c^{-1}(f)$  [2]. Thus the combination of channel and equalizer leads to a completely flat transfer function. This is an ideal equalizer, also known as zero-forcing (ZF) equalizer, since it forces the ISI to zero at the sampling instants  $t=kT$ ,  $k=0,1,\dots$ . As a result, the output of the ZF equalizer is expressed as follows

$$z_k = c_k + n_k, k=0, 1, \dots, n$$

where  $c_k$  is the desired symbol and  $n_k$  is the additive noise [4]. The ZF equalizer enhances the noise of the channel, at frequencies where the transfer function of the channel attains small values, since it performs channel inversion.

For the  $(2k+1)$ -tap equalizer, the value of  $k$  should satisfy  $2k+1 \geq L$  so that the equalizer spans the length of the Inter Symbol Interference, where  $L$  is the number of signal samples spanned by the ISI. The ZF condition to each sample at  $t=mT$  can be expressed as follows

$$c(mT) = \sum_{i=-K}^K w_i x(mT - i\tau) = \begin{cases} 1, m = 0 \\ 0, m = \pm 1, \dots, \pm K. \end{cases}$$

This is a set of  $2K+1$  linear equations for the coefficients  $w_i$  of the ZF equalizer. In matrix form it can be expressed as

$$\begin{aligned} X_w &= c \\ X_{ij} &= x(iT - j\tau), \\ i, j &= -K, -K+1, \dots, K, \\ w &= (w_{-K}, \dots, w_K)^T, \\ c &= (0, \dots, 0, 1, 0, \dots, 0)^T \end{aligned}$$

Thus the ZF solution can be expressed as follows

$$w = X^{-1}c$$

### II. BPSK Modulation Scheme

PSK is a modulation technique that conveys data by changing or modulating the phase of a reference signal called the carrier wave. The demodulator determines the phase of the received signal and maps it back the symbol it represents. The receiver compares the phase of the received signal to a reference signal; hence this system is termed coherent. PSK has a constant envelope, and this relaxes the requirements on the transmitter power amplifier. PSK is more bandwidth efficient than FSK, and more power efficient than ASK and FSK [5]. When the phase is differentially encoded, the demodulator then determines the changes in the phase of the received signal rather than the phase itself. This is differential phase-shift keying (DPSK). DPSK can be significantly simpler to implement than PSK since the demodulator does not need copy of the reference signal to determine the exact phase of the received signal, thus it is a non-coherent scheme. In return, it produces a

higher BEP at demodulation. When the communication channel introduces an arbitrary phase shift, the demodulator is unable to discriminate the constellation points; in this case, the data is often differentially encoded prior to modulation.

In BPSK, the carrier signal has constant amplitude but its phase is switched between two values, which are separated by pi, to represent 0 and 1 respectively. Typically, the two phases are 0 and  $\pi$ , and the signals are represented as follows

$$s_1, s_2(t) = \pm A \cos 2\pi f_c t, kT \leq t \leq (k+1)T$$

for 1 and 0 respectively. The signals are called antipodal, and they have a correlation coefficient  $\rho_{12} = -1$ , leading to the minimum BEP for a given  $\gamma_b = E_b / N_0$ . BPSK is a binary antipodal ASK. Demodulation of BPSK is coherent. The coherent detector can be in the form of either a correlator or a matched filter. A reference signal must be used in the receiver, which must be synchronous to the received signal in frequency and phase. A carrier recovery circuit can be used. At pass band, a correlator is typically used, since a matched filter with  $h(t) = \cos 2\pi f_c (T - t)$  is difficult to implement. In an uncoded AWGN channel, for coherent demodulation, BPSK has the BEP as a function of  $\gamma_b$  and is mathematically expressed for coherent BPSK as follows

$$P_b = Q(\sqrt{2\gamma_b})$$

The Power Spectral Density of the baseband BPSK signal is given as follows

$$\phi_s(f) = A^2 T \left( \frac{\sin \pi f T}{\pi f T} \right)^2$$

The null bandwidth  $B_{null} = 2R_b$ , where  $R_b$  being the data bit rate. The BPSK signals have a main lobe at the carrier frequency and many side lobes in the power spectrum. The use of raised cosine filtering can increase the spectral roll-off, and thus the spectral efficiency. This also makes the signal to no longer have a constant envelope. The raised cosine filtering method is applicable to QPSK.

Differentially encoded BPSK (DEBPSK) signals can be coherently demodulated or differentially demodulated. The PSD of the DEBPSK signal is the same as that of BPSK. The modulator and demodulator structures for various BPSK and DEBPSK schemes are given in. For DEBPSK signals that use differential modulation, the scheme is called differential BPSK (DBPSK) [6]. DBPSK does not use any coherent reference signal for demodulation, but uses the previous symbol as the reference for demodulating the current symbol. This leads to a suboptimum receiver. When DBPSK uses differential coherent demodulation, which requires a reference signal but does not require phase synchronization, we get the optimum DBPSK. The BEP of the optimum DBPSK is given by the following equation

$$P_b = \frac{1}{2} e^{-\gamma_b}$$

The suboptimum is used in practice as the DBPSK receiver. Its error performance is slightly inferior to that of the optimum case, with a loss smaller than 2 dB. When an ideal narrow-band IF filter with a bandwidth of  $W=0.57/T$  is placed before the correlator, the best performance is achieved with a loss of 1 dB in the case of Suboptimum DBPSK as follows

$$P_b = \frac{1}{2} e^{-0.8\gamma_b}$$

The DEBPSK signal can also be demodulated coherently, and this scheme is known as DEBPSK. It is the same as coherent BPSK, but uses differential encoding to eliminate phase ambiguity in the carrier recovery circuit for coherent PSK. The corresponding BEP is given by

$$P_b = 2Q(\sqrt{2\gamma_b}) \{1 - Q(\sqrt{2\gamma_b})\}$$

For large SNR, it is two times the Bit Error Probability of coherent BPSK without differential encoding.

#### IV. Performance of MIMO Systems with Different Zero Forcing Equalization Schemes:

In a 2 x 2 MIMO channel, the most probable usage of the available two transmit

antennas is having a transmission sequence of  $\{x_1, x_2, x_3, \dots, x_n\}$ . In normal transmission,  $x_1$  is sent in the first time slot,  $x_2$  in the second time slot and  $x_3$  in the third time slot and so on. However, as the number of transmit antennas is two, it is possible to group the symbols into groups of two. In the first time slot,  $x_1$  is sent and  $x_2$  is sent from the first and second antenna. In second time slot,  $x_3$  and  $x_4$  is sent from the first and second antenna and also  $x_5$  and  $x_6$  is sent in the third time slot and so on. It is observed that as the grouping of two symbols is done and then sent in one time slot, only  $n/2$  time slots to complete the transmission-data rate is doubled. This forms the simple explanation of a probable MIMO transmission scheme with 2 transmission and 2 receive antennas. The channel assumed to be a flat fading, that is, in simple terms; it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication. The channel experienced by each transmit antenna is independent from the channel experienced by other transmit antennas. For the  $i$ th transmit antenna to  $j$ th receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number. As the channel under consideration is a Rayleigh channel, the real and imaginary parts of are Gaussian distributed. The channel experienced between each transmit to the receive antenna is independent and randomly varying in time. On the receive antenna, the noise has the Gaussian probability density function and the channel too is known at the receiver. In classical Successive Interference Cancellation techniques, the receiver arbitrarily takes one of the estimated symbols, and subtracts its effect from the received symbols. To make that decision, finding out the transmit symbol which came at higher power at the receiver side is found out. Implementing Successive Interference Cancellation with optimal ordering ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower order probability than the other symbol. This results in lowering the chances of incorrect decisions resulting in erroneous

interference cancellation. Hence this gives lower error rate than simple successive interference cancellation.

Initially the random binary sequence of +1's and -1's is generated. It is then paired as two symbols and sent simultaneously in one time slot. Multiplication of the symbols with the channels is performed and then white Gaussian noise is added. Equalization of the received symbols is done. The hard decision decoding is implemented and the bit errors are counted. It is repeated for multiple values of  $E_b/N_0$  and finally the simulation results and theoretical results are plotted and compared. For Zero forcing equalizer with Successive Interference cancellation, after equalizing the received symbol with Zero Forcing criterion, the symbol having higher power is considered and then subtracted from the received signal. After this step, Maximal Ratio Combining (MRC) is performed for equalizing the new received symbol and then hard decision decoding is implemented followed by the count of bit errors and repeating it for multiple values of  $E_b/N_0$ . The plot results are analysed thoroughly. In case if Zero Forcing Successive Interference Cancellation with optimal ordering, after equalizing the received symbol with Zero Forcing criterion the power of received symbols from both the spatial dimensions is traced and then the simulation is carried out to obtain the theoretical and simulation results.

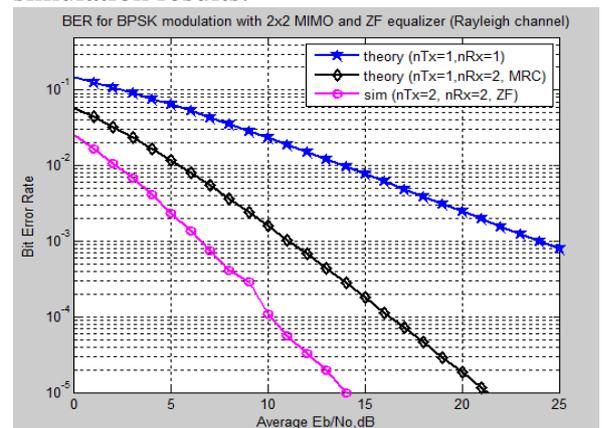


Fig. 1. Bit Error Rate for BPSK Modulation in 2 x 2 MIMO System using Zero Forcing Equalization

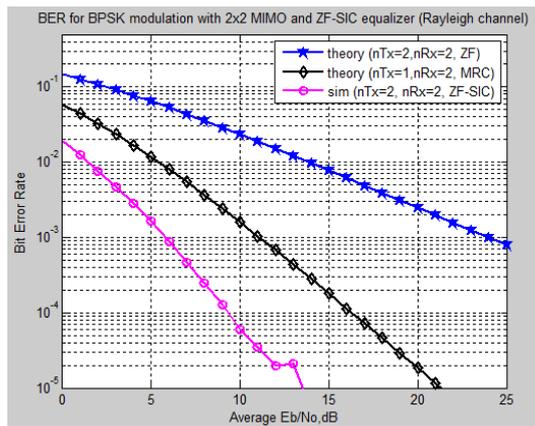


Fig. 2. Bit Error Rate for BPSK Modulation in 2 x 2 MIMO System using Zero Forcing with Successive Interference Cancellation

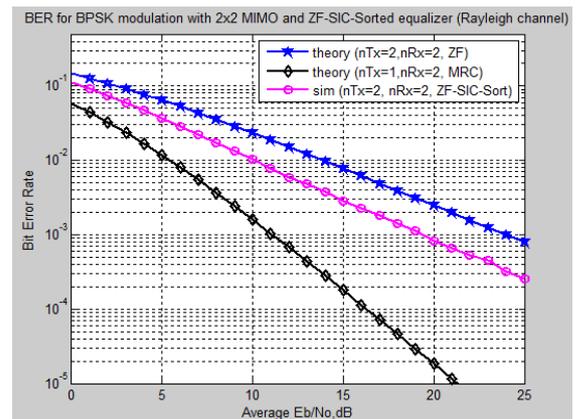


Fig. 3. Bit Error Rate for BPSK Modulation in 2 x 2 MIMO System using Zero Forcing with Successive Interference Cancellation and Optimal Ordering

### Conclusion

It is quite obvious from figures 1 and 2 that when the signal to noise ratio increases the Bit Error Rate gradually decreases. The Zero forcing with successive interference cancellation produces a very low bit error rate when compared to the standard Zero forcing Equalization technique. However from figure 3, it is evident that when optimal ordering is introduced along with Zero forcing successive interference cancellation technique the bit error rate is found to be high. Further works may include the analysis using different modulation schemes under different channel conditions to find out the lowest Bit Error Rate in different forms of Zero Forcing equalization scheme.

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