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Original Research Article

ONE RAISED PRODUCT PRIME LABELING OF SOME CYCLE RELATED GRAPHS

Sunoj B S[1*], Mathew Varkey T K[2]

[1] Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India
[2] Department of Mathematics, TKM College of Engineering, Kollam, Kerala, India

Abstract: One raised product prime labeling of a graph is the labeling of the vertices with {1,2---,p} and the edges with product of the labels of the incident vertices plus 1. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits one raised product prime labeling. Here we investigated some cycle related graphs for one raised product prime labeling.

Keywords: Graph labeling; product; prime labeling; prime graphs; cycle.

Introduction: All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.
A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In this paper we investigated one raised product prime labeling of some cycle related graphs.

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spalazhi@yahoo.com.
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Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (*gcin*) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

Main Results

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges .
Define a bijection f : V(G) → {1,2,---, p} by $f(v_i)$ = i , for every i from 1 to p and define a 1-1 mapping f^*_{orpp} : E(G) → set of natural numbers N by $f^*_{orpp}(uv) = f(u)f(v) + 1$.
The induced function f^*_{orpp} is said to be one raised product prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits one raised product prime labeling is called one raised product prime graph.

Theorem 2.1 Cycle C_n (n >2) admits one raised product prime labeling, if $(n+1) \not\equiv 0 \pmod 3$ and n is odd.

Proof: Let G = C_n and let $v_1, v_2, ---, v_n$ are the vertices of G.

Here $|V(G)| = n$ and $|E(G)| = n$.

Define a function $f : V \rightarrow \{1,2,---,n\}$ by

$$f(v_i) = i\ ,\ i = 1,2,---,n$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$$f^*_{orpp}(v_i\ v_{i+1}) = i^2+i+1 \qquad i = 1,2,---,n-1.$$

$$f^*_{orpp}(v_1\ v_n) = n+1.$$

Clearly f^*_{orpp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_1\ v_n)\}$

= gcd of {3, n+1}= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$

= gcd of $\{i^2+i+1, i^2+3i+3\}$

= gcd of $\{2i+2, i^2+i+1\}$

= gcd of {i+1,i(i+1)+1}

= 1, $\qquad$ i = 1,2,---,n-2.

gcin of (v_n) = gcd of $\{f^*_{orpp}(v_1\ v_n), f^*_{orpp}(v_{n-1}\ v_n)\}$

= gcd of $\{n+1, n^2-n+1\}$

= gcd of {3, n+1}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence C_n , admits one raised product prime labeling.

Example 2.1 G = C_7

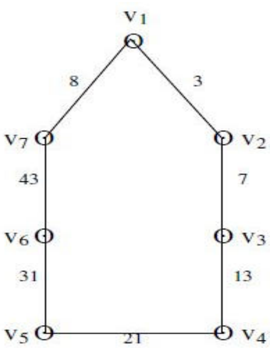

fig 2.1

Theorem 2.2 Let G be the graph obtained by duplicating one edge of cycle C_n (n >2) by a vertex. G admits one raised product prime labeling, if $(n+2) \not\equiv 0 \pmod 6$ and n is even.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{n+1}$ are the vertices of G.

Here $|V(G)| = n+1$ and $|E(G)| = n+2$.

Define a function $f : V \rightarrow \{1,2,---,n+1\}$ by

$$f(v_i) = i\ ,\ i = 1,2,---,n+1.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$$f^*_{orpp}(v_i\ v_{i+1}) = i^2+i+1, \qquad i = 1,2,---,n.$$

$$f^*_{orpp}(v_1\ v_3) = 4$$

$$f^*_{orpp}(v_1\ v_{n+1}) = n+2.$$

Clearly f^*_{orpp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_1\ v_3)\}$

= gcd of {3, 4}= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$= 1,

i = 1,2,---,n-1.

gcin of (v_{n+1}) = gcd of $\{f^*_{orpp}(v_1\ v_{n+1}), f^*_{orpp}(v_n\ v_{n+1})\}$= gcd of $\{n+2, n^2+n+1\}$

= gcd of {3, n+2}

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits one raised product prime labeling.

Example 2.2 Let G be the graph obtained by duplicating one edge of cycle C_6 by a vertex.

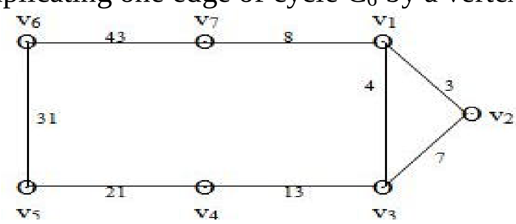

fig – 2.2

Theorem 2.3 Duplicating one vertex of cycle C_n (n >2) admits one raised product prime labeling, if $(n-2) \not\equiv 0 \pmod{13}$ and n is odd.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{n+2}$ are the vertices of G.

Here |V(G)| = n+2 and |E(G)| = n+3.

Define a function f : V → {1,2,---,n+2} by

$f(v_i) = i$, i = 1,2,---,n+2.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{i+1})$ = i(i+1)+1,

i = 1,2,---,n+1.

$f^*_{orpp}(v_1\ v_3)$ = 4

$f^*_{orpp}(v_3\ v_{n+2})$= 3n+7

Clearly f^*_{orpp} is an injection.

gcin of (v_1)= gcd of {$f^*_{orpp}(v_1\ v_2)$, $f^*_{orpp}(v_1\ v_3)$}= gcd of {3, 4}= 1.

gcin of (v_{i+1})= gcd of {$f^*_{orpp}(v_i\ v_{i+1})$, $f^*_{orpp}(v_{i+1}\ v_{i+2})$}= 1,

i = 1,2,---,n.

gcin of (v_{n+2}) = gcd of {$f^*_{orpp}(v_3\ v_{n+2})$, $f^*_{orpp}(v_{n+1}\ v_{n+2})$}= gcd of {3n+7, n^2+3n+3}

= gcd of {3n+7, n^2-4}

= gcd of {n-2, 3n+7}

= gcd of {n-2, 13}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits one raised product prime labeling.

Example 2.3 Let G be the graph obtained by duplicating one vertex of cycle C_7 by an edge.

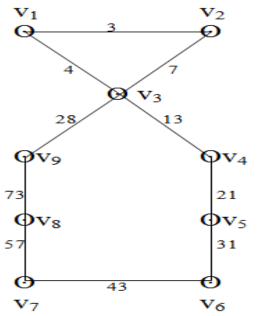

fig 2.3

Theorem 2.4 Let G be the graph obtained by joining path P_m to cycle C_n. G admits one raised product prime labeling, if (n+1) ≢ 0 (mod3) and n is odd.

Proof: Let G be the graph and let $v_1,v_2,---,v_{n+m-1}$ are the vertices of G.

Here |V(G)| = n+m-1 and |E(G)| = n+m-1.

Define a function f : V → {1,2,---,n+m-1} by

$f(v_i) = i$, i = 1,2,---,n+m-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{i+1})$ = i(i+1)+1,

i = 1,2,---,n+m-2.

$f^*_{orpp}(v_1\ v_n)$ = n+1.

Clearly f^*_{orpp} is an injection.

gcin of (v_1) = gcd of {$f^*_{orpp}(v_1\ v_2)$, $f^*_{orpp}(v_1\ v_n)$}

= gcd of {3, n+1}= 1.

gcin of (v_{i+1}) = gcd of {$f^*_{orpp}(v_i\ v_{i+1})$, $f^*_{orpp}(v_{i+1}\ v_{i+2})$}= 1,

i = 1,2,---,n+m-3.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits one raised product prime labeling.

Example 2.4 Let G be the graph obtained by joining path P_4 to cycle C_7.

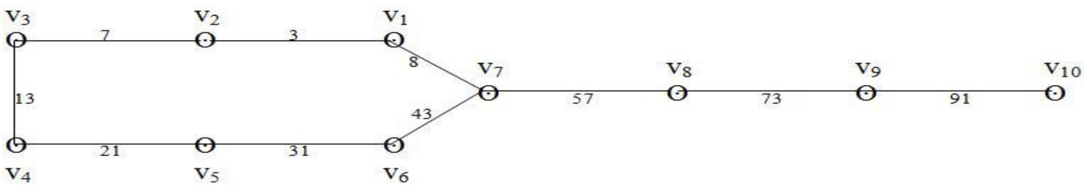

fig – 2.4

Theorem 2.5 Let G be the graph obtained by joining two copies of path P_m to two consecutive vertices cycle C_n. G admits one raised product prime labeling, if m and n are odd.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{n+2m-2}$ are the vertices of G.

Here |V(G)| = n+2m-2 and |E(G)| = n+2m-2.

Define a function f : V → {1,2,---,n+2m-2} by

$$f(v_i) = i \,,\ i = 1,2,---,n+2m-2$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^{*}_{orpp} is defined as follows

$f^{*}_{orpp}(v_i\, v_{i+1})$ = i(i+1)+1,

i = 1,2,---,n+2m-3.

$f^{*}_{orpp}(v_m\, v_{m+n-1})$ = m(m+n-1)+1.

Clearly f^{*}_{orpp} is an injection.

gcin of (v_{i+1}) = gcd of {$f^{*}_{orpp}(v_i\, v_{i+1})$, $f^{*}_{orpp}(v_{i+1}\, v_{i+2})$}= 1,

i = 1,2,---,n+2m-4.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits one raised product prime labeling.

Example 2.5 Let G be the graph obtained by joining two copies of path P_5 to two consecutive vertices cycle C_5.

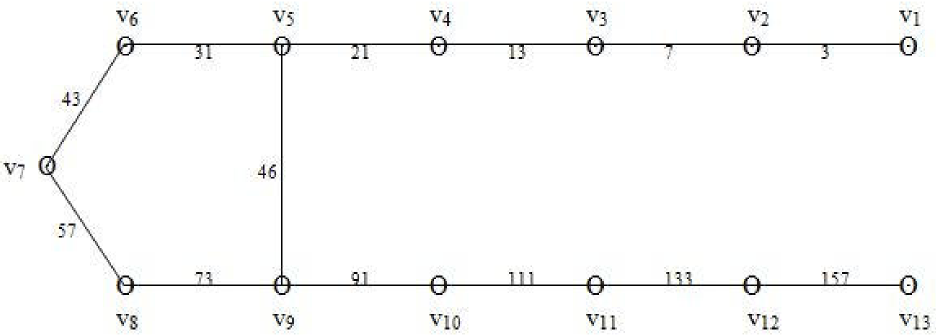

fig 2.5

Theorem 2.6 Let G be the graph obtained by joining the apex vertex of star $K_{1,n}$ to any one vertex of cycle C_n. G admits one raised product prime labeling, if (n+1)≢ 0(mod3) and n is odd.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n.

Define a function f : V → {1,2,---,2n} by

$$f(v_i) = i \,,\ i = 1,2,---,2n.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^{*}_{orpp} is defined as follows

$f^{*}_{orpp}(v_i\, v_{i+1})$ = i^2+i+1,

i = 1,2,---,n-1.

$f^{*}_{orpp}(v_1\, v_n)$ = n+1

$f^{*}_{orpp}(v_n\, v_{n+i})$= n^2+ni+1,

i = 1,2,---,n.

Clearly f^{*}_{orpp} is an injection.

gcin of (v_1)= gcd of {$f^{*}_{orpp}(v_1\, v_2)$, $f^{*}_{orpp}(v_1\, v_n)$}

= gcd of {3, n+1}= 1.

gcin of $(v_{i+1}$ =gcd of {$f^{*}_{orpp}(v_i\, v_{i+1})$, $f^{*}_{orpp}(v_{i+1}\, v_{i+2})$}= 1,

i = 1,2,---,n-1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits one raised product prime labeling.

Example 2.6 Let G be the graph obtained by joining the apex vertex of star $K_{1,n}$ to any one vertex of cycle C_7

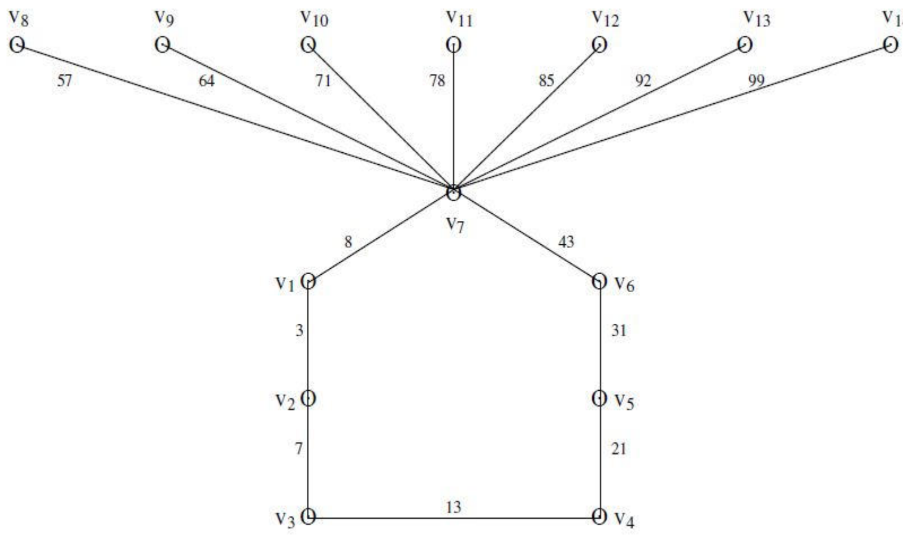

fig – 2.6

Theorem 2.7 Let G be the graph obtained by joining cycle C_3 to each vertex of path P_n . G admits one raised product prime labeling.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{3n}$ are the vertices of G.

Here |V(G)| = 3n and |E(G)| = 4n-1.

Define a function f : V → {1,2,---,3n} by

$f(v_i) = i$, i = 1,2,---,3n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{3i-2}\ v_{3i-1})= 9i^2-9i+3$, i = 1,2,---,n.

$f^*_{orpp}(v_{3i-2}\ v_{3i})= 9i^2-6i+1$, i = 1,2,---,n.

$f^*_{orpp}(v_{3i-1}\ v_{3i})= 9i^2-3i+1$, i = 1,2,---,n.

$f^*_{orpp}(v_{3i-1}\ v_{3i+2})= 9i^2+3i-1$, i = 1,2,---,n-1.

Clearly f^*_{orpp} is an injection.

gcin of (v_{3i-2}) = gcd of $\{f^*_{orpp}(v_{3i-2}\ v_{3i-1}), f^*_{orpp}(v_{3i-2}\ v_{3i})\}$

= gcd of $\{9i^2-9i+3, 9i^2-6i+1\}$

= gcd of $\{3i-2, 9i^2-9i+3\}$

= gcd of { 3i-2, (3i-2)(3i-1)+1}

= 1,

i = 1,2,---,n.

gcin of (v_{3i-1}) = gcd of $\{f^*_{orpp}(v_{3i-2}\ v_{3i-1}), f^*_{orpp}(v_{3i-1}\ v_{3i})\}$

= gcd of $\{9i^2-9i+3, 9i^2-3i+1\}$

= gcd of $\{6i-2, 9i^2-9i+3\}$

= gcd of { 3i-1, (3i-2)(3i-1)+1}

= 1,

i = 1,2,---,n.

gcin of (v_{3i}) = gcd of $\{f^*_{orpp}(v_{3i-2}\ v_{3i}), f^*_{orpp}(v_{3i-1}\ v_{3i})\}$

= gcd of $\{9i^2-6i+1, 9i^2-3i+1\}$

= gcd of $\{3i, 9i^2-6i+1\}$

= gcd of { 3i, (3i-2)(3i)+1}

= 1,

i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G, admits one raised product prime labeling.

Example 2.7 Let G be the graph obtained by joining cycle C_3 to each vertex of path P_3

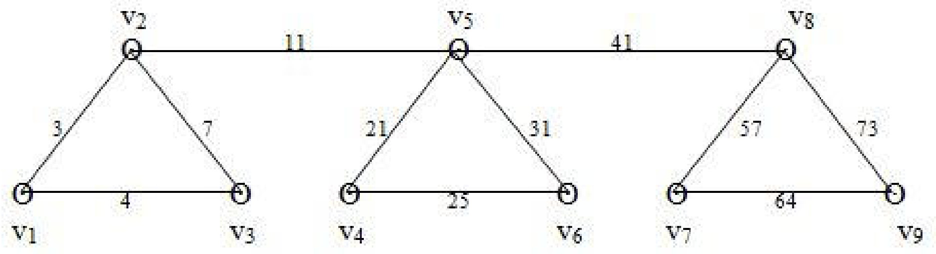

Fig – 2.7

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