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Original Research Article

S-PATH DOMINATION IN SHADOW DISTANCE GRAPHS

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Abstract: Let G = (V,E) be a simple connected and undirected graph. A subset D of V is called a dominating set of G if every vertex not in D is adjacent to some vertex in D. The domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A dominating set of G is called a s-path dominating set of G $(3 \le s \le diamG)$ if every path of length s in G has at least one vertex in this dominating set. We denote a s-path dominating set by D_{p_s}. The s-path domination number of G denoted by $\gamma_{p_s}(G)$ is the minimal cardinality taken over all s - path dominating sets of G. In this paper, we determine s - path domination number of the shadow distance graph of the path graph with specified distance sets.

Keywords: - Dominating set, vertex domination number, s-path domination number, Minimal vertex dominating set.

Introduction: By a graph $G=(V,E)$ we mean a finite undirected graph without loops and multiple edges. A subset D of V is called a dominating set of G if every vertex not in D is adjacent to some vertex in D. The domination number or vertex domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A vertex v in a graph G dominates the vertices in its closed neighbourhood N(v), that is, v is said to dominate itself and each of its neighbours.

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A dominating set of G is called a s-path dominating set of G $(3 \le s \le diamG)$ if every path of length s in G has atleast one vertex in this dominating set. We denote a s-path dominating set by D_{p_s}.The s-path domination number of G denoted by $\gamma_{p_s}(G)$ is the minimal cardinality taken over all s-path dominating sets of G. By definition every s-path dominating set is a dominating set but the converse is not true. Also it follows that $|D| \le |D_{p_s}|$ and hence $|\gamma(G)| \le |\gamma_{p_s}(G)|$.

Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u,v) \in D_s$.

The shadow distance graph of G, denoted by $D_{sd}(G, D_s)$ is constructed from G with the following conditions:

i) consider two copies of G say G itself and G'
ii) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u' \in V(G')$ (second copy)
iii) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G')$
iv) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G') \cup E_{ds}$ where E_{ds} is the set of all edges between two distinct vertices $u \in V(G)$ and $v' \in V(G')$ that satisfy the condition $d(u,v) \in D_s$ in G.

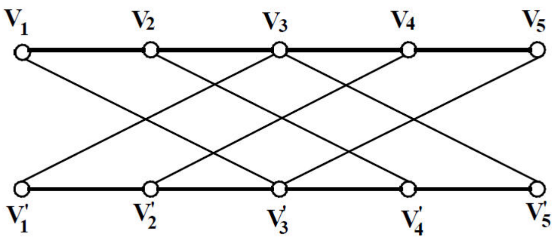

Figure 1. The graph $D_{sd}(P_5, \{2\})$

Main Results

Theorem 2.1. If G is a graph with no isolated vertices, then $\gamma(G) \leq \gamma_{p_3}(G) \leq \frac{n}{2}$

Proof: Let D_{p_s} is a minimal dominating set of G. Every vertex in D_{p_s} adjacent with at least one vertex in V- D_{p_s}. Hence V- D_{p_s} is a dominating set and $\gamma(G) \leq \gamma_{p_s}(G) \leq \min\left\{\left|D_{p_s}\right|, \left|V - D_{p_s}\right|\right\} \leq \frac{n}{2}$.

Theorem 2.2. For any graph G,

$$\gamma(G) \leq \gamma_{p_s}(G) \leq \left\lceil \frac{n+1-(\delta(G)-1)\frac{\Delta(G)}{\delta(G)}}{2} \right\rceil$$

Proof : The upper bound is immediate.

Theorem 2.3. For any graph G, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma(G) \leq \gamma_{p_s}(G)$

Proof: Let D_{p_s} be s-path dominating set of G. Each vertex dominates at most itself and $\Delta(G)$ other vertices. Hence the result.

The following results are immediate from the definition

Theorem 2.4. Let $n \geq 3$. Then $\gamma_{p_s}(P_n) = \left\lceil \frac{n}{3} \right\rceil, 3 \leq s \leq diam P_n$

We recall the following result related to $\gamma(G)$.

Theorem 2.5. [5] A dominating set D is a minimal dominating set if and only if for each vertex v in D, one of the following condition holds:

i) v is an isolated vertex of D
ii) there exists a vertex u $\in$ V-D such that N(u) $\cap$ D = $\{v\}$

An analogous result related to s-path domination is stated below;

Theorem 2.9. A dominating set D_{p_s} is a minimal dominating set if and only if for each vertex v in D_{p_s}, one of the following condition holds:

i) v is an isolated vertex of D_{p_s}
ii) there exists a vertex u $\in$ V- D_{p_s} such that N(u) $\cap$ D_{p_s} = $\{v\}$

We first provide below the results for vertex domination number of the shadow distance graph of the path graph with specified distance sets.

Theorem 2.10. Let $n \geq 5$. Then

$$\gamma(D_{sd}\{P_n, \{2\}\}) = 2\left\lceil \frac{n}{5} \right\rceil.$$

Proof : Consider two copies of P_n, one P_n itself and other denoted by $P_n^{'}$. Let $v_1, v_2, ...v_n$ be the vertices of P_n and let $v_1^{'}, v_2^{'}, ...v_n^{'}$ be the vertices of $P_n^{'}$. Let $e_1, e_2, ...e_{n-1}$ be the edges of the first copy P_n and $e_1^{'}, e_2^{'}, ...e_{n-1}^{'}$ be the edges of the second copy $P_n^{'}$, where $e_i = (v_i, v_{i+1}), e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for i = 1,2,…n-1.

Let $G = (D_{sd}\{P_n, \{2\}\})$.

Then $|V(G)| = 2n, |E(G)| = 4n - 6$ and

$E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{j,\{j+2\}^{'}}\} \cup \{e_{k,\{k-2\}^{'}}\}$

where $1 \le i \le n-1, 1 \le j \le n-2, 3 \le k \le n$.

Let $n \ge 6$.

Consider the set D = $V_1 \cup V_2$ where

$$V_1 = \{v_{5i-2}\} \cup \{v_{5i-2}^{'}\}, 1 \le i \le \left\lceil \frac{n}{5} \right\rceil - 1,$$

$$V_2 = \begin{cases} \{v_n, v_n^{'}\}, & n \equiv 1,2,3 (\text{mod } 5) \\ \{v_{n-1}, v_{n-1}^{'}\}, & n \equiv 4 (\text{mod } 5) \\ \{v_{n-2}, v_{n-2}^{'}\}, & n \equiv 0 (\text{mod } 5) \end{cases}$$

This set D is a minimal dominating set with minimum cardinality since for any vertex v ∈ D , D- {v} is not a dominating set. Thus, some vertex u in V-D is not dominated by any vertex in D∪ {v}. Now either u=v or u ∈ V-D. If u=v, then v is an isolated vertex of D. If u∈ V-D and u is not dominated by D - {v}, but is dominated by D, then u is adjacent only to vertex v in D, i.e N(v) ∪ D = {v}.

This implies that the set D described above is of minimum cardinality and since

$|D| = 2\left\lceil \frac{n}{5} \right\rceil$, it follows that

$$\gamma(D_{sd}\{P_n, \{2\}\}) = 2\left\lceil \frac{n}{5} \right\rceil.$$

Theorem 2.11. Let $n \ge 5$. Then

$$\gamma(D_{sd}\{P_n, \{3\}\}) = 2\left\lceil \frac{n+2}{5} \right\rceil.$$

Proof : Let $G = (D_{sd}\{P_n, \{3\}\})$ We consider the vertex set of G as in Theorem 2.10. and edge set

$E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{j,\{j+3\}^{'}}\} \cup \{e_{\{k-3\}^{'},k}\}$

where $1 \le i \le n-1, 1 \le j \le n-3, 1 \le k \le n$. Clearly $|V(G)| = 2n, |E(G)| = 4n - 8$.

Let $n \ge 5$.

Consider the set D = $V_1 \cup V_2$ where

$$V_1 = \{v_{5i-3}\} \cup \{v_{5i-3}^{'}\}, 1 \le i \le \left\lceil \frac{n-3}{5} \right\rceil,$$

$$V_2 = \begin{cases} \{v_n, v_n^{'}\}, & n \equiv 2,3,4 (\text{mod } 5) \\ \{v_{n-1}, v_{n-1}^{'}\}, & n \equiv 0,1 (\text{mod } 5) \end{cases}$$

This set D is a minimal dominating set with minimum cardinality since for any vertex v ∈ D , D- {v} is not a dominating set. Thus, some vertex u in V-D is not dominated by any vertex in D∪ {v}. Now either u=v or u ∈ V-D. If u=v, then v is an isolated vertex of D. If u∈ V-D and u is not dominated by D - {v}, but is dominated by D, then u is adjacent only to vertex v in D, i.e N(v) ∪ D = {v}.

This implies that the set D described above is of minimum cardinality and since

$|D| = 2\left\lceil \frac{n+2}{5} \right\rceil$, it follows that $\gamma(D_{sd}\{P_n, \{3\}\})$ = $2\left\lceil \frac{n+2}{5} \right\rceil$.

Hence the proof.

Theorem 2.12. Let $n \ge 5$. Then

$$\gamma_{p_3}((D_{sd}\{P_n, \{2\}\})) = \begin{cases} 4, & n = 5 \\ 6, & n = 6,7 \\ 2\left\lceil \frac{n}{2} \right\rceil - 2, & n \ge 8 \end{cases}$$

Proof : Let $G = (D_{sd}\{P_n, \{2\}\})$. We consider the vertex set and edge set of G are as in Theorem 2.10.

For n=5, the set D_{p_3} = $\{v_3, v_4, v_3^{'}, v_4^{'}\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =4.

For n=6, the set D_{p_3} = $\{v_3, v_4, v_6, v_3^{'}, v_4^{'}, v_6^{'}\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

For n=7, the set D_{p_3} = $\{v_3, v_4, v_7, v'_3, v'_4, v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

For n=8, the set D_{p_3} = $\{v_3, v_4, v_7, v'_3, v'_4, v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

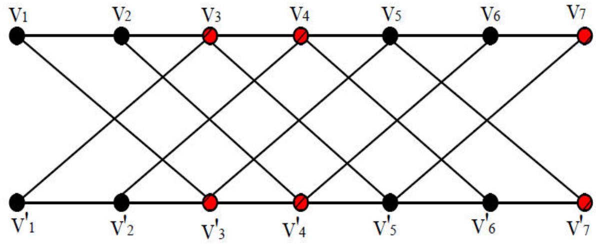

Figure 2. The graph $\gamma_{p_3}(D_{sd}(P_7,\{2\}))=6$

Let $n \geq 9$.

Consider the set D_{p_3} =

$$\begin{cases} \{v_{4j-1}\}\cup\{v_{4j}\}\cup\{v'_{4j-1}\}\cup\{v'_{4j}\}, & n\equiv 1,2(\bmod 4) \\ \{v_{4j-1}\}\cup\{v_n\}\cup\cup\{v_{4j}\}\cup\{v'_{4j-1}\}\cup\{v'_n\}\cup\{v'_{4j}\}, & n\equiv 3(\bmod 4) \\ \{v_{4j-1}\}\cup\{v_{n-1}\}\cup\cup\{v_{4j}\}\cup\{v'_{4j-1}\}\cup\{v'_{n-1}\}\cup\{v'_{4j}\}, & n\equiv 0(\bmod 4) \end{cases}$$

$$\text{where} \begin{cases} 1\leq j\leq \left\lfloor \frac{n}{4} \right\rfloor, & n\equiv 1,2(\bmod 4) \\ 1\leq j\leq \left\lfloor \frac{n}{4} \right\rfloor, & n\equiv 3(\bmod 4) \\ 1\leq j\leq \frac{n}{4}-1, & n\equiv 0(\bmod 4) \end{cases}$$

This set D_{p_3} is aminimal dominating sets with minimum cardinality since for any vertex v ∈ D_{p_3} , D_{p_3} - {v} is not a 3-path dominating set. Thus, some vertex u in V-D_{p_3} is not dominated by any vertex in D_{p_3} ∪ {v}. Now either u=v or u ∈ V-D_{p_3}. If u=v, then v is an isolated vertex of D_{p_3}. If u∈ V-D_{p_3} and u is not dominated by D_{p_3} - {v}, but is dominated by D_{p_3}, then u is adjacent only to vertex v in D_{p_3},i.e N(v) ∪ D_{p_3} = {v}.

This implies that the set D_{p_3} described above is of minimum cardinality and since $|D_{p_3}|=2\left\lceil \frac{n}{2} \right\rceil-2$ it follows that $\gamma_{p_3}((D_{sd}\{P_n,\{2\}\}))= 2\left\lceil \frac{n}{2} \right\rceil-2$.

Hence the proof.

Theorem2.13. Let n≥5.Then $\gamma_{p_3}(D_{sd}\{P_n,\{3\}\})$

$$= \begin{cases} 4, & n=5 \\ 6, & n=6 \\ 2\left\lceil \frac{n}{2} \right\rceil, & n\geq 7 \end{cases}$$

Proof: : Let $G=(D_{sd}\{P_n,\{3\}\})$ We consider the vertex set and edge set of G are as in Theorem 2.11.

For n=5, the set D_{p_3} = $\{v_2, v_4, v'_2, v'_4\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =4.

For n=6, the set D_{p_3} = $\{v_2, v_4, v_5, v'_2, v'_4, v'_5\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

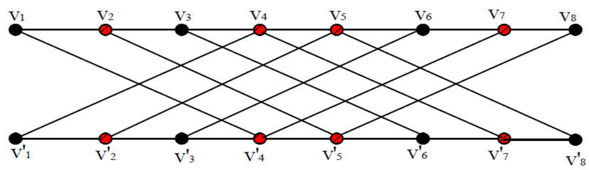

Figure 3. The graph $\gamma_{p_3}(D_{sd}(P_8,\{3\}))=8$ Consider

Let $n \geq 7$.

$$D_{p_3} = V_1 \cup V_2,\ where\ V_1=\{v_2,v_4,v'_2,v'_4\}, V_2=\{v_{2j+3}\}\cup\{v'_{2j+3}\}, 1\leq j \leq \left\lceil \frac{n}{2} \right\rceil - 2$$

This set D_{p_3} is aminimal dominating sets with minimum cardinality since for any vertex v ∈ D_{p_3} , D_{p_3} - {v} is not a 3-path dominating set. Thus, some vertex u in V- D_{p_3} is not dominated by any vertex in D_{p_3} ∪ {v}. Now either u=v or u ∈ V- D_{p_3}. If u=v, then v is an isolated vertex of D_{p_3}. If u∈ V- D_{p_3} and u is not dominated by D_{p_3} - {v}, but is dominated by D_{p_3}, then u is adjacent only to vertex v in D_{p_3}, i.e N(v) ∪ D_{p_3} = {v}.

This implies that the set D_{p_3} described above is of minimum cardinality and since $|D_{p_3}| = 2\left\lceil \frac{n}{2} \right\rceil$ it follows that $\gamma_{p_3}(D_{sd}\{P_n,\{3\}\}) = 2\left\lceil \frac{n}{2} \right\rceil$.

Hence the proof.

Theorem2.14. $\gamma_{p_4}((D_{sd}\{P_n,\{2\}\})) =$

$$\begin{cases} \left\lceil \frac{n}{3} \right\rceil, & 6 \leq n \leq 10 \\ 2\left\lfloor \frac{n+(2j+2)}{3} \right\rfloor, & 6j+5 \leq n \leq 7j+10,\ j \geq 1 \end{cases}$$

Proof: Let $G=(D_{sd}\{P_n,\{2\}\})$. We consider the vertex set and edge set of G are as in Theorem 2.10.

For n=6, the set D_{p_4} = $\{v_3,v_4,v'_3,v'_4\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$=4.

For n=7, the set D_{p_4} = $\{v_3,v_4,v_7,v'_3,v'_4,v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$=6.

For n=8, the set D_{p_4} = $\{v_3,v_4,v_7,v'_3,v'_4,v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$=6.

For n=9, the set D_{p_4} = $\{v_3,v_4,v_7,v'_3,v'_4,v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$=6.

For n=10, the set D_{p_4} = $\{v_3,v_4,v_7,v_{10},v'_3,v'_4,v'_7,v'_{10}\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$=8.

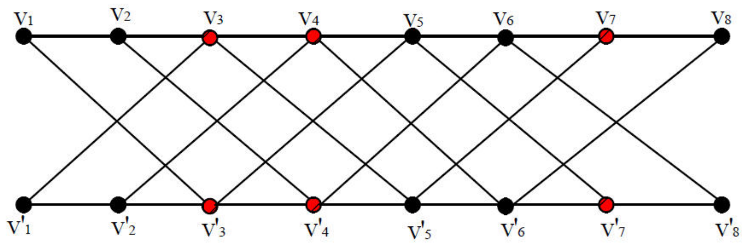

Figure 4. The graph $\gamma_{p_4}(D_{sd}(P_8,\{2\}))=6$

Let $n \geq 11$.

Consider $D_{p_4} = V_1 \cup V_2 \cup V_3$,.

where

$$V_1=\{v_{7j-4},v_{7j-3}\}\cup\{v'_{7j-4},v'_{7j-3}\}\cup\{v_n,v'_n\}\cup\{v_{7j},v'_{7j}\},\ n\equiv 3(\text{mod}\,7),1\le j\le\left\lfloor\frac{n}{7}\right\rfloor$$

$$V_2=\{v_{7i-4},v_{7i-3}\}\cup\{v'_{7i-4},v'_{7i-3}\}\cup\{v_{7j},v'_{7j}\},n\equiv 0,1,2(\text{mod}\,7),1\le i\le\left\lfloor\frac{n}{7}\right\rfloor$$

$$V_3=\{v_{7k-4},v_{7k-3}\}\cup\{v'_{7k-4},v'_{7k-3}\}\cup\{v_{7k},v'_{7k}\},n\equiv 4,5,6(\text{mod}\,7),1\le k\le\left\lceil\frac{n}{7}\right\rceil$$

This set D_{p_4} is aminimal dominating sets with minimum cardinality since for any vertex v $\in D_{p_4}$, D_{p_4} - {v} is not a 4-path dominating set. Thus, some vertex u in V- D_{p_4} is not dominated by any vertex in $D_{p_4} \cup \{v\}$. Now either u=v or u $\in$ V- D_{p_4}. If u=v, then v is an isolated vertex of D_{p_4}. If u$\in$ V- D_{p_4} and u is not dominated by D_{p_4} - {v}, but is dominated by D_{p_4}, then u is adjacent only to vertex v in D_{p_4},i.e N(v) $\cup$ D_{p_4} = {v}.

This implies that the set D_{p_4} described above is of minimum cardinality and since

$$|D_{p_4}|= 2\left\lfloor\frac{n+(2j+2)}{3}\right\rfloor,\ 6j+5\le n\le 7j+10,\ j\ge 1,$$

it follows that

$$\gamma_{p_4}((D_{sd}\{P_n,\{2\}\}))= 2\left\lfloor\frac{n+(2j+2)}{3}\right\rfloor,\ 6j+5\le n\le 7j+10,\ j\ge 1.$$

Hence the proof.

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